Int. J. Heat Mass Transfer. Vol. 11, pp. 883–897. Pergamon Press 1968. Printed in Great Britain

A NUMERICAL EXPERIMENT OF RADIANT HEAT INTERCHANGE BY THE MONTE CARLO METHOD

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(*Received* 14 *August* 1967 *and in revised form* 20 *November* 1967)

Abstract—Radiant heat interchange between simply arranged surfaces having direction dependent characteristics has been studied by the statistical method known as Monte Carlo. The diffuse and specular constant property, the specular directional emission and reflection, and the directional emission and the bidirectional reflection models have been explored with the preliminary goal of understanding what level of detail is important in the description of the radiation properties of surfaces. Four simple configurations which permit a critical examination of the directional effects have been examined. Results showed that under some conditions the choice of the model for radiation surface characteristics can be very critical for both the local radiant heat flux and for overall radiant interchange calculations.

NOMENCLATURE

A,	area;
a_0,	autocorrelation distance;
B_{ij},	overall absorption factor defined by equation (4);
B_{di-j},	local absorption factor defined by equation (3);
BP,	refers to directional emission and bidirectional reflection surface property model;
CP,	constant (direction independent) properties;
D,	diffuse;
DP,	directional properties;
E,	emitted energy flux;
$\mathscr{F}$,	Hottel's radiation exchange factor;
f,	reflection distribution function;
I_b,	intensity of blackbody radiation given by Planck's function;
k,	imaginary part of complex index of refraction;
L, W, H, Z,	dimensions of surfaces, see Fig. 2;
N,	number of energy bundles emitted;
$\tilde{n}$,	complex index of refraction;
n,	index of refraction;
Q,	radiant heat-transfer rate;
$Q_{i\rightleftarrows j}$,	radiant heat interchange rate between surfaces i and j;
q,	radiant heat flux;
R,	random number, see equation (6);
S,	specular;
T,	temperature;
x, y, z,	coordinates, see Fig. 2.

Greek symbols

α,	absorptivity of a surface;
β,	included angle, see Fig. 2;
ϵ,	emissivity of a surface;
ρ,	reflectivity of a surface;
θ,	polar angle;
λ,	wavelength of radiation;
σ,	Stefan–Boltzmann constant;
σ_0,	root mean square roughness;
ϕ,	azimuthal angle;
Ω,	solid angle.

Superscripts

d,	diffuse component of reflectivity;
s,	specular component of reflectivity;

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refers to the incident direction.

INTRODUCTION

AN UNDERSTANDING of radiant heat transfer has become increasingly important for modern day technology. Contemporary applications include space vehicles capable of penetrating unfamiliar thermal environments, solar energy conversion devices, power plants for space exploration needs, long-life cryogenic storage tanks, propulsion systems, industrial thermal systems, and others. In these applications radiation heat transfer may be the major or the only means of energy transfer and its determination is of considerable practical importance.

Some of the newer applications have called for radiant heat-transfer predictions of higher precision and greater detail than were considered necessary in the past. This demand has provided the recent impetus for the research effort in the various aspects of radiant heat transfer and has resulted in a number of new approaches for computation of radiation interchange among surfaces. A number of simplified models for approximating the radiation surface properties have been suggested, and the book by Sparrow and Cess [1] furnishes an up to date survey of applications and calculations. Unfortunately, the validity of these models has not been substantiated by more realistic and refined analyses. It is therefore essential to know what errors are introduced by these computational procedures.

The purpose of this paper is therefore to examine the validity of the commonly used simplified models and to determine what level of detail is important in describing the radiation characteristics of real surfaces when predicting overall radiant heat transfer and interchange rates or the local radiant heat flux. The rigorous solution of the exactly formulated radiation interchange problem in an enclosure having direction and wavelength dependent properties presents formidable mathematical difficulties and is impractical for engineering calculations [2]. At the present time the only method powerful enough for numerical computation of radiation exchange in real enclosures appears to be the Monte Carlo method which is adopted in this paper.

The Monte Carlo method simulates directly a physical problem by means of statistical sampling. The method is conceptually very simple and is particularly well-adopted to deal with complex geometrical configurations and direction dependent radiation characteristics of surfaces. The method has been applied to radiation interchange problems by a number of investigators [3–5], to mention just a few. A more complete bibliography is given elsewhere [6].

ANALYSIS

General formulation of the problem

To define the scope of this paper, the following radiant interchange problem is posed. Given an enclosure consisting of a finite number of isothermal surfaces, predict the overall radiant heat-transfer rate and the local radiant heat flux. The temperatures and the radiation properties of the surfaces are considered to be arbitrary but are assumed to be specified.

Rather than discuss the general problem, we choose to impose certain constraints which make it more tractable. The following simplifying assumptions are employed in the analysis.

1. The geometric optics theory is valid for the analysis of radiant heat transfer. The two fundamental postulates of the theory are: (1) the various effects and quantities are additive, and (2) the directional change of a pencil of radiation is negligible. The former assumption implies that the phenomena of diffraction, interference and coherence are excluded [7].
2. Planck's and Kirchhoff's laws are valid.
3. The radiation characteristics of surfaces are independent of the polarization state of the incident intensity.

4. The medium separating the surfaces does not participate in the exchange of radiation, and the index of refraction of the medium is constant, i.e. equal to unity.

That assumption 3 is not generally valid is well recognized [7] and has been examined in more detail by Edwards and Bevans [8]. Assumptions 1 and 2 are not expected to seriously impair the relevancy of the analysis for engineering applications.

The Monte Carlo method

The Monte Carlo method is in essence a statistical method which can be simply interpreted in terms of physical processes. The method takes the Lagrangian point of view by considering individual histories of "energy bundles" or photons from the point of emission to point of absorption or escape from the system. The integral equation formulation of radiation interchange among surfaces on the other hand, takes the Eulerian point of view by focusing attention on the directional distribution of intensity of radiation incident or leaving a particular location. The method is conceptually very simple and can be extended to very general situations. The details pertinent to the Monte Carlo techniques are well known [9] and an excellent review of recent applications pertinent to radiation interchange is given by Howell [3]. For definiteness only the basic concepts relating to the present problem will be introduced.

The energy emitted by a surface is divided into a number of equal energy parts called "energy bundles". Application of the Monte Carlo method consists of emitting these energy bundles in directions from a surface (or an elementary area) proportional to the actual energy emitted from the surface in those directions. These energy bundles then play the game of chance according to the actual deterministic and random features of the physical processes step by step [9]. In other words, the energy bundles are followed and the events in their life history are noted until the energy bundles are absorbed or escapes from the system. The directions of the energy bundles are modified by the surfaces of the system according to the actual reflection, transmission, etc. characteristics. After tracing the histories of a sufficiently large number of bundles and summing (or averaging) the events one can determine what fraction of the emitted energy has been absorbed and reflected at each surface or has escaped from the system through an opening.

Consider radiation interchange in an enclosure consisting of n number of real or imaginary surfaces and direct attention to the emission of radiation from an elementary area $\mathrm{d}A_i$ located on surface i at a point denoted by the position vector $\mathbf{r}_i$. According to the definition of the emitted flux, the amount of radiant energy emitted from $\mathrm{d}A_i$ (see Fig. 1) is

$$E_i(\mathbf{r}_i)\,\mathrm{d}A_i = \Big[\int_0^\infty \int_{A_i} \epsilon_i(\mathbf{r}_i, \theta_i, \phi_i, \lambda)\, I_{bi}(\mathbf{r}_i, \lambda) \cos\theta_i\,\mathrm{d}\Omega_i\,\mathrm{d}\lambda\Big]\,\mathrm{d}A_i. \quad (1)$$

If this emitted energy is subdivided into $N_{\mathrm{d}i}$ individual "energy bundles" or "energy packets" then the energy per bundle is given by

$$e_i = E_i\,\mathrm{d}A_i/N_{\mathrm{d}i}. \quad (2)$$

If out of the total number $N_{\mathrm{d}i}(= N_i\,\mathrm{d}A_i/A_i)$ of energy bundles emitted $N_{\mathrm{d}i\to j}$ are eventually absorbed at surface j directly or after interreflections, the absorption factor $B_{\mathrm{d}i-j}$ can be expressed as

$$B_{\mathrm{d}i-j} = \lim_{N_{\mathrm{d}i}\to\infty} (N_{\mathrm{d}i\to j}/N_{\mathrm{d}i}). \quad (3)$$

The factor $B_{\mathrm{d}i-j}$ is a generalization of the absorption factor introduced by Gebhart [10]. The absorption factor between two finite surfaces is defined in an analogous manner,

$$B_{ij} = \lim_{N_i\to\infty} (N_{i\to j}/N_i) \quad (4)$$

where $N_{i\to j}$ represents the number of energy bundles emitted by a finite surface i which are eventually absorbed at surface j. The

absorption factors B_{ij} are simply related to the Hottel's radiation exchange factors $\mathscr{F}_{ij}$ [11] by

$$\mathscr{F}_{ij} = \epsilon_i B_{ij} \tag{5}$$

where ϵ_i is the total hemispherical emissivity of surface i. The absorption factors arise more naturally in the calculations than the radiation exchange factors and therefore the results are presented in terms of the former.

range 0–1 [4, 6]. If the bundle is absorbed then the energy is added to the amount appropriate to the particular surface or subdivision if local values are desired. If it is reflected the direction of the reflected bundle is determined from the reflection characteristics of the surface at that point. For example, the cumulative distribution function for determining the polar angle of the reflected energy bundle is given by

$$R_{\theta_i}(\theta_i', \phi_i', \lambda) = \frac{\int_0^{\theta_i}\int_0^{2\pi} f_i(\theta_i', \phi_i', \theta_i, \phi_i, \lambda)\cos\theta_i \sin\theta_i \, d\phi_i \, d\theta_i}{\int_0^{\pi/2}\int_0^{2\pi} f_i(\theta_i', \phi_i', \theta_i, \phi_i, \lambda)\cos\theta_i \sin\theta_i \, d\phi_i \, d\theta_i} \tag{6}$$

The procedure for tracing the histories of N_i energy bundles requires that each is emitted from the surface i in directions (θ_i, ϕ_i) in such a manner that these N_i energy bundles are representative of the actual energy emitted from the surface i. Also the wavelength labels attached to energy bundles should represent the actual energy emitted in these wavelength intervals and in these directions. This can be done by forming a cumulative distribution function for given emission characteristics of the surface [3].

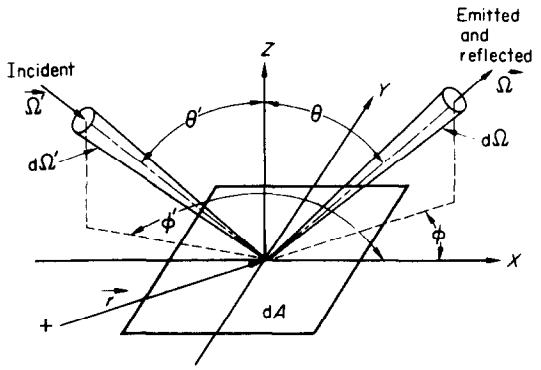

FIG. 1. Geometry of radiation incident and leaving a differential area.

After the energy bundle is emitted from some point on surface i in some direction its point and angle of incidence on surface k (say) is determined. The decision whether this energy bundle is absorbed, transmitted or reflected is made by generating a random number R in the and corresponding to this value of θ_i the azimuthal angle ϕ_i of reflected energy bundle is calculated in the same way. After determining the direction of reflection, the energy bundle is again followed until it is absorbed at the surface or leaves the enclosure.

Formation of other cumulative distribution functions needed in the calculations is discussed in detail elsewhere [6], and expressions for the limiting cases of diffuse emission and diffuse and specular reflection from gray surfaces are given in [3, 4, 6].

Models for radiation surface characteristics

In order to examine the validity of the commonly used simplified analyses and to estimate what level of detail is important in radiant interchange calculations, models for radiation surface characteristics which are physically more realistic are needed. The following models have been considered in the calculations:

A. Constant (direction independent) properties, *CP*.
   1. Diffuse emission and reflection with uniform radiosity, *D** (the results reported for this model were obtained analytically).
   2. Diffuse emission and reflection, *D*.
   3. Diffuse emission and specular reflection, *S*.

B. Direction dependent properties, *DP*.
   1. Directional emission and specular direction dependent reflection, *S*.

2. Directional emission and bidirectional reflection, $BP(\sigma_0/\lambda; a_0/\lambda)$.

The reason the diffuse emission, reflection and uniform radiosity model was also considered is that all of the zonal methods of radiant heat-transfer analysis such as the radiosity, Hottel, Gebhart and Oppenheim electric analog methods are based on this model.

The directional reflectivity $\rho(\theta', \lambda)$ was predicted from the Fresnel's equations in terms of the complex index of refraction, $\tilde{n}(\lambda) = n(\lambda) - ik(\lambda)$ [1, 7]. For opaque materials the spectral directional absorptivity is related to reflectivity by

$$\alpha(\theta', \lambda) = 1 - \rho(\theta', \lambda). \tag{7}$$

Furthermore, Kirchhoff's law

$$\epsilon(\theta', \lambda) = \alpha(\theta', \lambda) \tag{8}$$

permits one to infer spectral directional emissivity.

The reflection distribution function (bidirectional reflectance) $f(\theta', \phi'; \theta, \phi; \lambda)$ was approximated by

$$f(\theta', \phi'; \theta, \phi; \lambda) \simeq \rho(\theta', \lambda) f_\infty(\theta', \phi'; \theta, \phi; \lambda) \tag{9}$$

where $f_\infty(\theta', \phi'; \theta, \phi; \lambda)$ denotes Beckmann's reflection distribution function for a material having infinite electrical conductivity [12]. The reflection distribution function f_∞ depends on the optical roughness σ_0/λ and the optical auto-correlation distance a_0/λ. The basic postulates made by Beckmann in deriving the theoretical model for random rough surfaces are discussed in detail elsewhere [12, 13] and need not be repeated here. Recently Houchens and Hering [13] have critically examined Beckmann's model and have found it agreed reasonably well with the limited experimental data available.

It is recognized that the agreement between the predictions based on classical electromagnetic theory and data for $\rho(\theta', \lambda)$ is only qualitative; however, the directional trends are reasonably correct. In general equation (9) is also not expected to correctly account for finite conductivity of materials; however, the difficulty of including this effect rigorously in the theory [12] partly justifies this approximation. Furthermore, it should be remarked that the intent here has not been an accurate reproduction of directional radiation properties for a particular material, but only a qualitatively correct functional form which represents the gross trends of the characteristics with direction.

Method of solution

In the solution of a radiant interchange problem by the Monte Carlo method good quality random numbers are required. The speed of generation and cycle is also of considerable importance. The numbers generated by arithmetical processes are not generated in a random fashion but pass some criterion of randomness. The choice of a random number generator depends upon the particular problem and the computer used. A detailed discussion is given by Taussky and Todd [9]. The present problem was solved on an IBM 7094 computer, and the multiplicative congruential generator was used. It generates the random number sequence by taking the lower order thirty-six bits of the products $(x_i)5^{15}$ where x_i is the previous random number and $x_0 = 1$. The output is a number between 0 and 1.

The probability of accuracy P that the calculated result with sample size N lies within r per cent of exact value p is [14]

$$P \simeq \operatorname{erf}(\eta) \tag{10}$$

where

$$\eta = (r/100)[Np/2(1 - p)]^{\frac{1}{2}}. \tag{11}$$

This shows that for the same accuracy N should be large when p is small and vice versa; or with the same value of N larger values of result carry greater weight of confidence. Since the computer time is directly proportional to number of energy bundles traced, this method becomes very time consuming for small values of p. A number of shortcut methods to increase

the accuracy of the Monte Carlo method without increasing the sample size have been suggested by Toor [6].

In order to separate the directional from the spectral effects the surfaces were considered as gray, and the emission from the surface was taken to be uniform. For constant property models simple analytical relations exist between the cumulative distribution functions and azimuthal as well as the polar angles for emission and reflection. For direction dependent property models, the values of θ vs. R_θ for determining the direction of emission were stored in a form of a table in the memory of the computer and the values of θ were obtained by interpolation. For determining the direction of energy bundles reflected bidirectionally using the cumulative distribution function given by equation (6), for example, the values of θ vs. $R_\theta(\theta', \phi', \lambda)$ and ϕ vs. $R_\phi(\theta', \phi', \theta, \lambda)$ were calculated and stored in the computer memory at 5° step increments of the angle of incidence θ'. From this table the direction of the reflected energy bundles (θ, ϕ), corresponding to a given direction of incidence (θ', ϕ'), was obtained by interpolation.

Heat transfer

Once the absorption factors B_{ij} and B_{di-j} have been determined the overall heat-transfer rate at a surface, the overall radiant interchange between two surfaces, and the local radiant heat flux can be computed immediately for an enclosure consisting of isothermal surfaces. Recalling that the overall radiant heat-transfer rate from surface i can in general be expressed as the difference between energies emitted and absorbed at a surface, one can write

$$Q_i = A_i\epsilon_i(\sigma T_i^4 - \sum_{j=1}^{n} B_{ij}\sigma T_j^4) \qquad (12)$$

if use is made of reciprocity condition

$$A_i\epsilon_i B_{ij} = A_j\epsilon_j B_{ji} \qquad (13)$$

and if there is no radiation falling on the system from some external source. The net radiant interchange rate between two surfaces i and j is given in terms of the overall absorption factors B_{ij} as

$$Q_{i\rightleftarrows j} = A_iB_{ij}\epsilon_i(\sigma T_i^4 - \sigma T_j^4) = A_i\mathscr{F}_{ij}(\sigma T_i^4 - \sigma T_j^4). \qquad (14)$$

The local radiant heat flux at surface i can be expressed analogously to that by equation (12), and we can write

$$q_i = \mathrm{d}Q_i/\mathrm{d}A_i = \epsilon_i(\sigma T_i^4 - \sum_{j=1}^{n} B_{di-j}\sigma T_j^4) \qquad (15)$$

where in writing equation (15) use is made of the reciprocity condition

$$\epsilon_i \mathrm{d}A_i B_{di-j} = \epsilon_j A_j B_{j-di}. \qquad (16)$$

Various special cases follow immediately from equation (12) and (15). For example, when there are only two surfaces in the system and both are at the same temperature the local radiant heat loss,† equation (15) reduces to

$$q_i/\epsilon_i\sigma T_i^4 = 1 - B_{di-i} - B_{di-j}. \qquad (17)$$

RESULTS AND DISCUSSION

Independent parameters

Radiation heat exchange between surfaces depends on the geometry of the system, the radiation characteristics of surfaces and on the boundary conditions prescribed. There is, of course, an infinite number of combinations of the various independent parameters, and so it is necessary to be selective. The geometrical arrangements selected for study are illustrated in Fig. 2 and are henceforth referred to as configuration 1, configuration 2, etc. The choice of these simple configurations was dictated by the fact that such simply arranged surfaces arise in engineering systems. The systems chosen cover a range of geometries and as such permit a critical examination of the influence of directional characteristics of surfaces on radiation interchange.

The hemispherical emissivities adopted for

† When the temperature of the surfaces are identical the local radiant heat flux is referred to as local radiant heat loss.

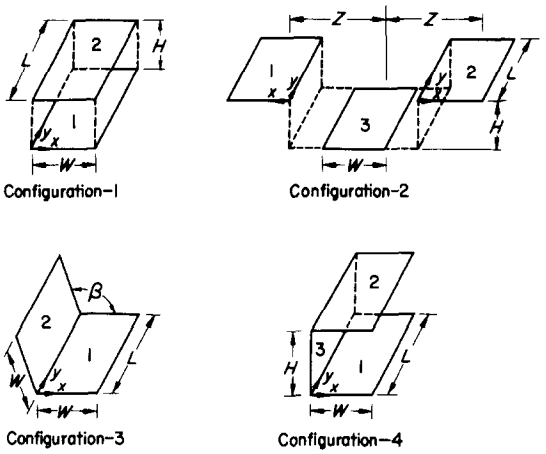

FIG. 2. Configurations analyzed.

the calculations were 0·9, 0·5 and 0·1. The reasons for choosing these values were twofold. First, they encompass a wide range of materials including poor as well as good electrical conductors. Second, these values were taken to facilitate direct comparison of the present results with those reported in the literature [15–18]. The complex index of refraction, $\tilde{n} = n - ik$, that is, the index of refraction n and the index of absorption k corresponding to these hemispherical emissivities are as follows:

ϵ	n	k
0·9	1·5565	0
0·5	6·1038	0
0·1	23·4520	23·4520

These constants are identical to those used by Hering [17].

For the sake of brevity, only sample results are included in this paper. Extensive tables of overall absorption factors and graphical results for local absorption factors and local heat loss are given in the thesis by Toor [6].

Comparison of Monte Carlo with exact results

The accuracy of the Monte Carlo method can be estimated and the validity of its use in radiant heat exchange calculations established by comparing the overall heat loss predictions with the exact results of Sparrow *et al.* [15] based on the diffuse model and the results of Hering [17] based on the specular model with constant (*CP*) and dependent properties (*DP*). Inspection of Table 1 reveals generally good agreement up to two significant figures. It should be remarked in passing that the exact results of Sparrow *et al.* [15] and Hering [17] were obtained for infinitely long plates, but the Monte Carlo calculations were based on length to width ratios $L/W = 5000$. These ratios are sufficiently large to accurately approximate the infinitely long plates. An equally good agreement has been obtained for configuration 1 with diffuse surfaces [6].

A comparison between the exact analytical predictions of Hering [18] and the approximate results based on the Monte Carlo method for the local heat loss are given in Fig. 3. This is a more severe test of the accuracy of the Monte Carlo method than has been provided by the comparison of the overall heat losses from the surface. The local heat flux predicted is a maximum of 3 per cent higher than the exact result. This small discrepancy establishes the utility of the Monte Carlo method in local radiant heat-transfer calculations.

It may be pointed out that when $B_{ii} + B_{ij}$ [equation (12)] is small compared to unity, errors in the absorption factors are masked when the results are presented in terms of the overall heat loss and it is only then that there is a greater probability of a larger percent error in the absorption factors.

Although the computer time taken to solve a problem for a specific configuration depends on the number of trials (probability of accuracy desired), the radiation characteristics, the computer program, etc., an idea of the computer time used can be obtained from the example given below. The calculation of the local absorption factors needed to predict the local heat loss results presented in Fig. 3 took about

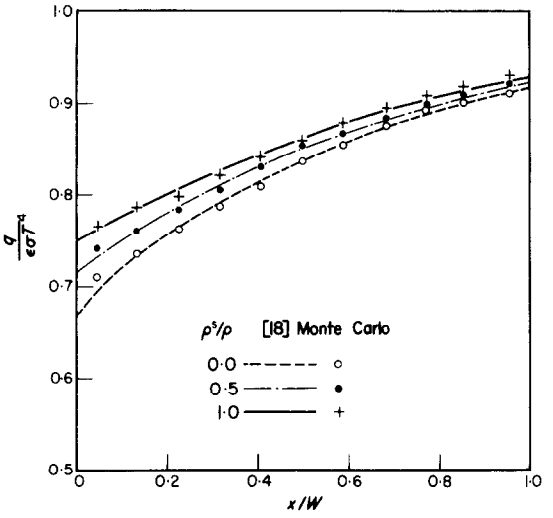

FIG. 3. Comparison of Monte Carlo and analytical [18] results for local heat loss for configuration 3; identical surfaces 1 and 2, $T_1 = T_2 = T$, $\epsilon_1 = \epsilon_2 = 0{\cdot}5$, $\beta = 90°$, $L/W = 5000{\cdot}0$.

3·5 min for the diffuse model and about 2·5 min for the specular model on the IBM 7094 computer. In both cases 50000 histories of energy bundles were traced. The method is more efficient for calculating overall absorption factors or heat transfer and considerably less efficient for predicting local values. For example, only 5000 histories of energy bundles were traced (for a given value of ϵ and β) to predict the overall heat loss results presented in Table 1.

Overall radiant heat transfer and radiant heat interchange

For the two limiting cases with $\epsilon = 1$ or $\epsilon = 0$ at all surfaces of the enclosure the different models for radiation characteristics are expected to yield the same prediction for radiant heat transfer since there are no interreflections in the former and no absorption or emission in the latter enclosure. The greatest difference between the various models is expected to occur when the reflectivity is high, say, $\epsilon = 0{\cdot}1$. This expectation is confirmed by the results shown in Fig. 4 for the overall radiation interchange factors. The results show that for $\epsilon = 0{\cdot}5$ the maximum difference between $\mathscr{F}_{12}$ predicted by any two models is less than 12 per cent; therefore, the overall absorption factors B_{12} and B_{11} for the four configurations given in Tables 2–5 are primarily for $\epsilon = 0{\cdot}1$. For the sake of brevity only sample results are presented here, but a complete tabulation is given elsewhere [6].

Examination of the results for the diffuse models with uniform (D^*) and nonuniform (D) radiosities presented in Table 2 shows good agreement. A similar finding has already been noted by Sparrow *et al.* [15] for infinitely long plates. The greatest difference between the results for the diffuse (D) and specular (S) constant property models occurs for "open"† configurations with highly reflecting surfaces. In some instances B_{12} based on the specular model may be greater by about a factor of five than those based on the diffuse model.

Comparison of the absorption factors for the specular model (S) based on the constant property (CP) and directional property (DP) analyses reveals that the agreement between the two sets of results is best, (see Tables 2 and 3) for "closed" configurations. The difference between the CP and DP results increases as the system becomes more "open" and in some instances the two sets of results differ by more than a factor of two.

For small values of σ_0/λ (i.e. $\sigma_0/\lambda = \frac{1}{30}$) the surface is effectively optically smooth [6] and it is not surprising that the results for the bidirectional reflection model (BP), (see Tables 2 and 4) are in good agreement with the results for the specular, direction dependent (DP) property model. The surface described by the optical roughness $\sigma_0/\lambda = \frac{2}{3}$ and the optical autocorrelation distance $a_0/\lambda = 10$ is rough and therefore approaches a diffuse reflection

† In the broad sense, the word "open" refers to such a system for which the ratio of heat loss to emission is approximately equal to unity. On the other hand, the word "closed" refers to a system for which this ratio is much smaller than one.

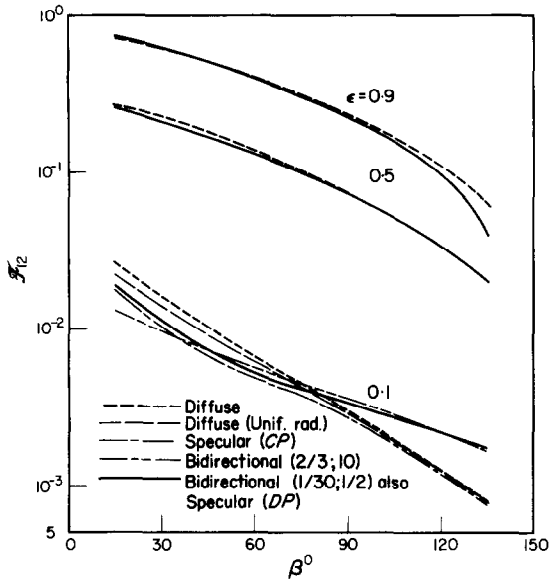

FIG. 4. Effect of emissivity on the overall radiation exchange factor $\mathscr{F}_{12}$ for configuration 3; identical surfaces 1 and 2, $L/W = 5000{\cdot}0$.

condition. Comparison of the results for this case with those for the diffuse, constant property model shows that the two sets of results differ in some instances by about a factor of two. The reason for this discrepancy is due to the dependence of radiation characteristics on direction for the bidirectional reflection model.

For closed and intermediate systems the overall heat transfer from a surface is not very sensitive to the choice of the model for the reflection characteristics as can be seen from the tabulated results. For example, since the average beam of radiation emitted from a surface, which is a part of a closed system, undergoes, say, more than ten reflections for a surface having $\epsilon = 0{\cdot}1$, its direction is practically random. In this case the probability that the beam will be absorbed by the surface is proportional to the absorptivity of the surface and is independent of the character of the reflections. However, for open systems with highly reflecting surfaces the overall heat transfer is much more sensitive to the nature of reflections since even for $\epsilon = 0{\cdot}1$ the beam undergoes few if any reflections at all. It is clear from equation (12) and the tabulated results that the largest difference between the overall heat-transfer predictions at surface i based on the various models occurs for open systems having highly reflecting surfaces when the temperature of surface j is larger than that of surface i, see Fig. 5.

Table 1. Comparison of overall heat loss from surface 1 (or 2) for configuration 3 with infinitely long plates, $T_1 = T_2 = T$ and $\epsilon_1 = \epsilon_2 = \epsilon$

		$Q/W\sigma T^4$			
	ϵ	0·9		0·1	
Model	β°	Monte Carlo	Exact	Monte Carlo	Exact
Specular	45	0·376	0·377	0·0895	0·0895
DP	90	0·681	0·677	0·0966	0·0965
[17]	135	0·861	0·858	0·0983	0·0983
Specular	45	0·376	0·376	0·0907	0·0906
CP	90	0·662	0·663	0·0971	0·0971
[17]	135	0·837	0·838	0·0993	0·0992
	45	0·363	0·365	0·0836	0·0838
Diffuse	90	0·658	0·658	0·0958	0·0958
[15]	135	0·836	0·836	0·0992	0·0992

Table 2. Overall absorption factors for configuration 1 with $L/W = 0.5$ and $\epsilon_1 = \epsilon_2 = \epsilon$

ϵ	H/W	CP			DP	BP(σ_0/λ; a_0/λ)	
		D^*	D	S	S	$\frac{2}{3}$; 10	$\frac{1}{30}$; $\frac{1}{2}$
	(a) B_{12}						
	2·0	0·0326	0·0334	0·0328	0·0356	0·0352	0·0357
	0·5	0·258	0·254	0·252	0·286	0·284	0·288
0·9	0·05	0·785	0·790	0·788	0·812	0·812	0·812
	0·005	0·895	0·895	0·893	0·911	0·912	0·912
	2·0	0·0036	0·0037	0·0048	0·0022	0·0025	0·003
	0·5	0·031	0·031	0·037	0·0243	0·0207	0·021
0·1	0·05	0·220	0·258	0·238	0·206	0·128	0·174
	0·005	0·461	0·489	0·461	0·463	0·403	0·454
	(b) B_{11}						
	2·0	0·00012	0·00013	0·0014	0·0015	0·0001	0·00047
	0·5	0·0079	0·0087	0·0074	0·0097	0·0035	0·0068
0·1	0·05	0·172	0·205	0·187	0·155	0·0835	0·130
	0·005	0·409	0·435	0·414	0·408	0·348	0·381

Table 3. Overall absorption factors B_{12} for configuration 2; surfaces 1 and 2 are diffuse and surface 3 is adiabatic and perfectly reflecting with $H/W = 1.0$ and $L/W = 1.0$

ϵ	Z/W	CP		BP(σ_0/λ; a_0/λ)		
		D	S	$\frac{2}{3}$; 10	$\frac{1}{15}$; 1	$\frac{1}{30}$; $\frac{1}{2}$
1·0	0·5	0·0239	0·050	0·0296	0·0352	0·0423
0·5	0·5	0·0122	0·0251	0·0156	0·0176	0·0213
0·1	0·5	0·00248	0·00528	0·00328	0·00358	0·00427
1·0	1·0	0·0058	0·0201	0·0156	0·0178	0·0186
0·5	1·0	0·0030	0·0103	0·00824	0·0085	0·0092
0·1	1·0	0·000458	0·00215	0·00152	0·00192	0·00195

The results presented in Tables 2–5 and Figs. 4 and 5 show that the choice of the model for radiation characteristics of surfaces is more critical in the prediction of overall radiant interchange between two surfaces than overall heat transfer at a particular surface. Only when the temperature of surface 1 is much lower than that of surface 2 is the choice of the model very important in predicting overall heat transfer at surface 1, see Fig. 5, but this is due to the fact that $Q_1 \simeq Q_{1\rightleftarrows 2}$ under these conditions. Finally, the choice of the model for predicting overall radiant interchange rate is more important for open than closed configurations.

Local radiant heat transfer

For the purpose of estimating the detail required in describing the radiation characteristics of surfaces when predicting the radiant heat flux the local absorption factors were calculated for a range of parameters and the four configurations considered. Some representative absorption factors are illustrated in Fig. 6. The results are presented in terms of

Table 4. Overall absorption factor for configuration 3 with $L/W = 1{\cdot}0$ and $\epsilon_1 = \epsilon_2 = 0{\cdot}1$

$\beta°$	CP		DP		$BP(\sigma_0/\lambda; a_0/\lambda)$	
	D^*	S	D	S	$\frac{2}{3}; 10$	$\frac{1}{30}; \frac{1}{2}$
(a) B_{12}						
15		0·143	0·208	0·141	0·107	0·144
45	0·060	0·051	0·070	0·051	0·051	0·050
60	0·0418	0·0327	0·0464	0·0339	0·0374	0·0346
90	0·0207	0·0189	0·0206	0·0208	0·0216	0·0203
135	0·0048	0·0044	0·0049	0·0090	0·0092	0·0094
(b) B_{11}						
15		0·0973	0·162	0·0973	0·064	0·102
45	0·025	0·0181	0·0389	0·0194	0·0258	0·0197
60	0·014	0·0071	0·0202	0·0119	0·0188	0·0115
90	0·0037	0·0000	0·0054	0·0000	0·00809	0·00072
135	0·00036	0·0000	0·0002	0·0000	0·00296	0·000061

Table 5. Overall absorption factors for configuration 4; surface 3 is adiabatic, diffuse and perfect reflector, identical surfaces 1 and 2, $\epsilon_1 = \epsilon_2 = 0{\cdot}1$, $L/W = 5000{\cdot}0$

H/W	CP		DP	$BP(\sigma_0/\lambda; a_0/\lambda)$	
	D	S	S	$\frac{2}{3}; 10$	$\frac{1}{30}; \frac{1}{2}$
(a) B_{12}					
4·0	0·032	0·022	0·018	0·014	0·015
1·0	0·123	0·086	0·070	0·114	0·067
0·25	0·255	0·202	0·179	0·285	0·188
(b) B_{11}					
4·0	0·014	0·022	0·022	0·018	0·022
1·0	0·047	0·060	0·052	0·095	0·049
0·25	0·209	0·157	0·136	0·240	0·146

the factor

$$B_{j-\mathrm{d}i}(A_j/\mathrm{d}A_i).$$

This is advantageous since this factor is independent of the size of the elementary area used in the numerical calculations. It should be noted that the results presented are not truly local but represent values averaged over a finite area ΔA_i. Since there was some scatter in the results the lines drawn through the points represent smoothed values.

Some representative results for the local radiant heat loss are presented in Figs. 7–9. The local absorption factors needed to predict the local radiant heat loss as well as additional results can be found in the thesis by Toor [6]. For configuration 1 local heat fluxes have been reported by Sparrow *et al.* [15] for diffuse surfaces and by Eckert and Sparrow [16] for specular surfaces having constant properties and by Hering [17] for direction dependent properties. The dependence of the local heat loss for "open", "closed" and "intermediate" geometries has been systematically discussed

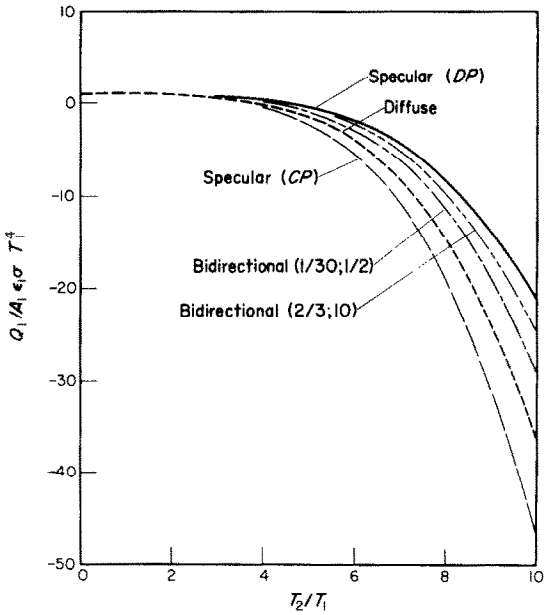

FIG. 5. The dependence of the overall radiant heat-transfer rate on the temperature ratio of the two surfaces for configuration 1; $\epsilon_1 = \epsilon_2 = 0{\cdot}1$, $L/W = 0{\cdot}5$, $H/W = 2{\cdot}0$.

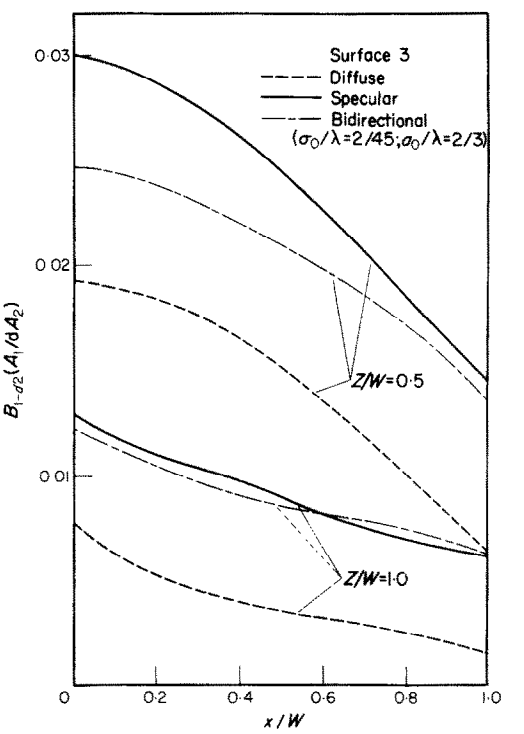

FIG. 6. Local absorption factor for configuration 2; diffuse surfaces 1 and 2, $\epsilon_1 = \epsilon_2 = 0{\cdot}1$, $\rho_3 = 1{\cdot}0$, $H/W = 1{\cdot}0$, $L/W = 5000{\cdot}0$.

by Sparrow *et al.* and needs no elaboration here.

The local heat loss is most nonuniform for black surfaces ($\epsilon = 1{\cdot}0$) and becomes uniform for the limiting case of perfectly reflecting surfaces ($\rho \to 1$). For very closed systems, e.g. configuration 1 where $(H/W) \to 0$, the ratio of heat loss at the edge of the plates to that at the center would be effectively infinite. For open geometries the local heat loss from a surface is practically uniform over the surface and the presence of a second or additional surface plays a very minor role. As the system becomes more closed the local heat loss becomes more nonuniform.

Based on the results presented in the figures and others [6] it is clear that the greatest difference between the predictions for $q/\epsilon\sigma T^4$ based on constant property models occurs at a position where the local heat loss is the smallest. The values differ in some instance, for example in Fig. 8, by about a factor of two. Furthermore, the choice of the model is most critical and the greatest level of detail is required

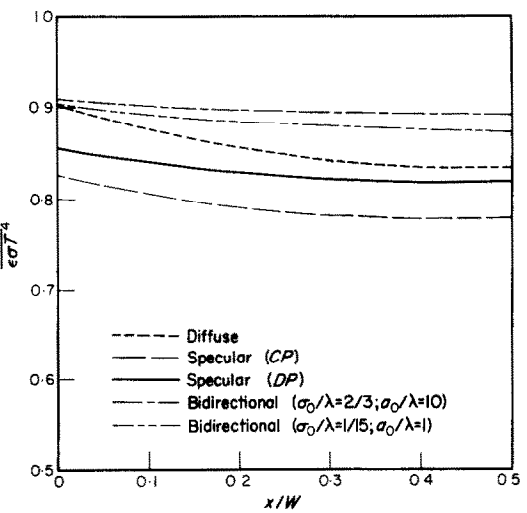

FIG. 7. Local heat loss for configuration 1; identical surfaces. $T_1 = T_2 = T$, $\epsilon_1 = \epsilon_2 = 0{\cdot}1$, $H/W = 0{\cdot}5$, $L/W = 2500{\cdot}0$.

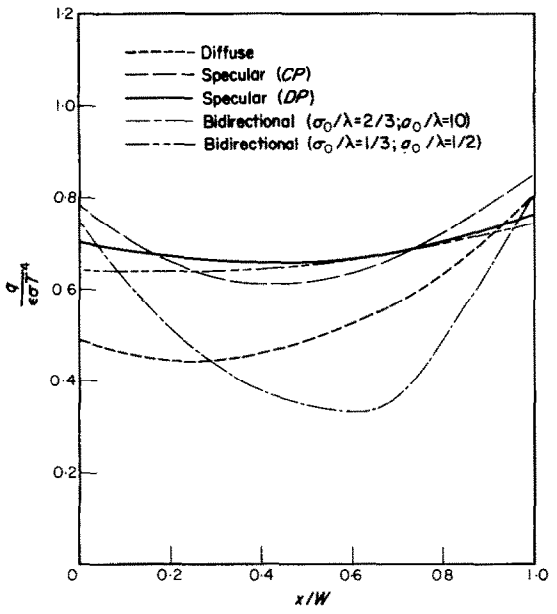

FIG. 8. Local heat loss for configuration 4; surface 3 is adiabatic, diffuse and perfect reflector, identical surfaces 1 and 2, $T_1 = T_2 = T$, $\epsilon_1 = \epsilon_2 = 0{\cdot}1$, $H/W = 0{\cdot}25$, $L/W = 5000{\cdot}0$.

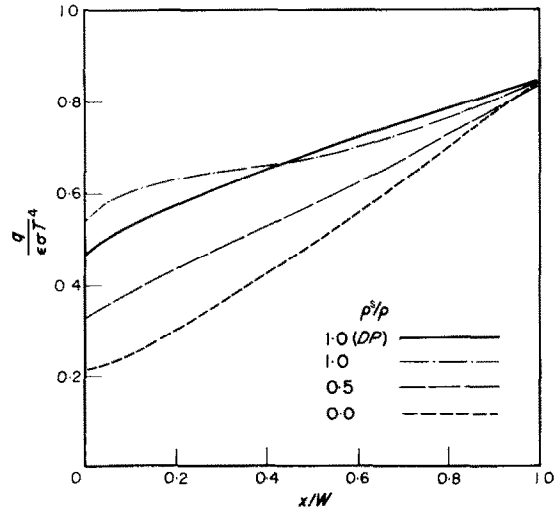

FIG. 9. Local heat loss for configuration 3; identical surfaces 1 and 2, $T_1 = T_2 = T$, $\epsilon_1 = \epsilon_2 = 0{\cdot}1$, $\beta = 15°$, $L/W = 5000{\cdot}0$.

in describing the radiation characteristics when calculating local radiant heat loss at a highly reflecting surface which is a part of a closed system.

Although the dimensionless local heat loss $q/\epsilon\sigma T^4$ is a quantity of interest it does not appear to provide a critical comparison of the different models, see Fig. 7, since the results based on various models do not differ by more than 10 per cent. The local radiant interchange $Q_{di\rightarrow j}$, i.e., the net radiant exchange between an elementary area dA_i and a finite area A_j provides a much more critical check of the validity of the model, particularly for open configurations.

CONCLUSIONS

Based on the Monte Carlo calculations of radiant interchange among surfaces the following conclusions may be drawn:

1. For complex geometries or when the radiation characteristics of surfaces depend on direction, Monte Carlo simulation of radiation exchange problems is attractive. The desired results can be expressed as overall and local absorption or radiation interchange factors or overall and local heat flux distributions on the surface. The method yields results that converge exactly (in the limit of many trials) to those of the specified physical problem; however, for very open systems such as configuration 2 when Z/W is large, the method is very inefficient.
2. The choice of the model and the detail needed to describe the radiation characteristics of participating surfaces of a given system is more important in analyzing radiation interchange between two surfaces than predicting overall radiant heat transfer at a surface. The choice of the model for radiation properties of surfaces is more critical for predicting overall radiant heat interchange in an open geometry than for a closed one, and the opposite is true for predicting the radiant heat transfer at a surface.
3. The choice of the model and the level of detail needed is more important for predicting local radiant interchange than local radiant

heat flux. In predicting the local radiant interchange the choice of the model is most critical for open geometries having highly reflecting surfaces, but in calculating the local radiant heat flux the choice of the model is most critical for closed geometries having highly reflecting surfaces.

4. The simple diffuse and specular constant property models can fail badly for a particular geometry and surface condition when the variation of properties with direction is not taken into account.
5. From the practical viewpoint, the directional emission and bidirectional reflection model for radiant heat-transfer analysis is too complicated for use in engineering calculations. Even for the very simple configurations considered very extensive computations are needed and do not appear to be justifiable at the present time in view of the fact that the detailed surface property information required to implement the real property analysis is generally not available for engineering materials.
6. An excellent compromise between reality, level of detail (where absolutely necessary) and computational effort appears to be the directional emission and reflection model with diffuse and specular components of reflectance such that

$$\rho(\theta', \lambda) = \rho^d(\theta', \lambda) + \rho^s(\theta', \lambda).$$

7. The Monte Carlo method can readily be extended with some additional complexity for calculation of total radiant heat transfer on the spectral basis including the polarization effects. There is great incentive to streamline the method and to make it more efficient and economical for radiant heat-transfer calculations and more work needs to be done in this area.

ACKNOWLEDGEMENTS

The work described in this paper was supported by the Air Force Flight Dynamics Laboratory, U.S. Air Force, under Contract No. AF 33(615)-2362. The Technical Monitor was Mr. Carl J. Feldmanis.

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Résumé—L'échange de chaleur par rayonnement entre des surfaces arrangées de façon simple et ayant des caractéristiques dépendant de la direction a été étudié par la méthode statistique connue sous le nom de Monte Carlo. Les modèles d'émission et de réflexion diffuse et spéculaire à propriétés constantes, et spéculaire directionnelle ont été examinés dans le but préliminaire de comprendre quel niveau de détail est important dans la description des propriétés de rayonnement des surfaces. Quatre configurations simples qui permettent un examen critique des effets directionnels ont été examinées. Les résultats montrent que, sous certaines conditions, le choix du modèle pour les caractéristiques de la surface rayonnante peut être très critique à la fois pour le flux local de chaleur par rayonnement et pour les calculs de l'échange global par rayonnement.

Zusammenfassung—Der Wärmeaustausch durch Strahlung zwischen einfach angeordneten Oberflächen mit richtungsabhängiger Charakteristik wurde mit der als Monte Carlo bekannten statistischen Methode untersucht. Dabei wurden folgende Modelle durchgerechnet, mit dem Ziel, herauszufinden, welche Einzelheiten für die Beschreibung der Strahlungseigenschaften von Oberflächen wichtig sind: (1) Richtungsabhängige Strahlung, sowohl diffus wie auch spiegelnd; (2) Spiegelnde, richtungsabhängige Emission und Reflexion; (3) Richtungsabhängige Emission und zweiseitig gerichtete Reflexion. Vier einfache Konfigurationen, welche eine kritische Untersuchung der Richtungseffekte gestatten, wurden behandelt. Die Ergebnisse zeigten, dass unter gewissen Voraussetzungen die Wahl des Modells für die Oberflächen-Strahlungscharakteristik sehr entscheidend sein kann und zwar sowohl für den lokalen Strahlungs-Wärmestrom wie auch für die Berechnung des Gesamtwärmeaustausches durch Strahlung.

Аннотация—С помощью статистического метода, известного под названием метода Монте Карло, изучался лучистый теплообмен между просто расположенными поверхностями при зависящих от направления характеристиках переноса. Были исследованы модели с диффузными и зеркальными постоянными свойствами, зеркальной направленной эмиссией и отражением для того, чтобы получить представление о том, какие детали являются важными при описании радиационных свойств поверхностей. Исследовались четыре простые конфигурации, позволяющие критически подойти к рассмотрению эффектов направленности излучения. Результаты показали, что при определенных условиях для расчетов как локального лучистого теплового потока, так и полного лучистого теплообмена выбор модели для излучательных характеристик поверхности может быть весьма важным.